Algorithmic Game Theory - handout2

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It you have the library copy of [NRTV], please return it to the library as soon as possible.

Homework. (Please keep the answers short and easy to read.)

Reading: you are assumed to know basic notions in computational complexity (definitions of classical complexity classes such as P, NP, PSPACE and EXPTIME, the notions of *hardness* and *completeness*). If needed, refresh your memory on these concepts. See for example [Papadimitriou].

In class we showed the well known algorithm for finding a stable matching (a.k.a. stable marriage) of Gale and Shapely [GS62]. This algorithm is also presented in Chapter 10 in [NRTV], in Wikipedia, and elsewhere.

Consider the following game with $2n$ players, n boys and n girls, each having his/her own preference list over partners of the other sex. In this game, every boy and every girl supplies a preference list (either their true preference list, or some other preference list), the outcome of the game is the matching produced by the stable matching algorithm when run on the supplied preference lists (the algorithm where the unengaged boys propose), and the payoff for a player is the rank (in the player's list) of the partner assigned to the player. An interesting question is whether the players have incentives to play *truthfully* in this game. Namely, is it always to the benefit of a player to report his or her true preference list, or may the player win a better partner (from the player's point of view) by reporting a different preference list?

1. We remarked (without proof) in class that the algorithm is optimal for the boys. Prove that for every boy, reporting his true preference list is a dominant strategy. (Hint. Show that to match a boy with a higher ranked girl starts off a chain of changes to the existing matching, in which each boy in the chain gets a higher ranked girl with respect to his preferences, until a cycle is closed. Show that this cycle certifies that the existing matching could not have been the outcome of the algorithm.)

2. Show that all players following the strategy of reporting their true preference lists is not necessarily a Nash equilibrium of the game. Namely, show an example ($n = 3$ suffices for this purpose), where a girl can benefit (eventually be matched by the algorithm to a boy that she prefers more) by reporting a preference list that is different from her true preference list.

3. Prove that this game always has some pure Nash equilibrium (though as question 2 shows, in this Nash equilibrium some players might not be reporting their true preferences).

References

[GS62] D. Gale and L. S. Shapley: College Admissions and the Stability of Marriage, American Mathematical Monthly 69, 9-14, 1962.

[Papadimitriou] Christos Papadimitriou. Computational Complexity. Addison-Wesley. 1994.

[NRTV] Noam Nisan, Tim Roughgarden, Eva Tardos and Vijay V. Vazirani (Editors), Algorithmic Game Theory, Cambridge University Press, 2007.