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Atmospheric and Oceanic Fluid Dynamics

Fundamentals and Large-Scale Circulation

Second Edition

The atmosphere and ocean are two of the most important components of the climate system, and fluid dynamics is central to our understanding of both. This book provides a unified and comprehensive treatment of the field that blends classical results with modern interpretations. It takes the reader seamlessly from the basics to the frontiers of knowledge, from the equations of motion to modern theories of the general circulation of the atmosphere and ocean. These concepts are illustrated throughout the book with observations and numerical examples. As well as updating existing chapters, this second, full-colour edition includes new chapters on tropical dynamics, El Niño, the stratosphere and gravity waves. Supplementary resources are provided online, including figures from the book and problem sets, making this new edition an ideal resource for students and scientists in the atmospheric, oceanic and climate sciences, as well as in applied mathematics and engineering.

Geoffrey K. Vallis is a professor of applied mathematics at the University of Exeter, UK. Prior to taking up his position there, he taught for many years at Princeton University in the USA. He has carried out research in the atmospheric sciences, oceanography and the planetary sciences, and has published over 100 peer-reviewed journal articles. He is the recipient of various prizes and awards, including the Adrian Gill Prize (Royal Meteorological Society) in 2014, and the Stanislaw M. Ulam Distinguished Scholar award (Los Alamos National Laboratory) in 2013.

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"Vallis writes explanations as clear as tropical ocean waters, bringing fresh new light to complex concepts. This expanded text will be immediately useful both for graduate students and seasoned researchers in the field."

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"In 2006, Vallis' first edition of AOFD offered the atmospheric and oceanic sciences community a truly great book, marking a milestone in our discipline. Well, Vallis has done it again! This second edition of AOFD represents the pinnacle of a maturing discipline. It is **The Great Book** of the field, and it will remain so for a generation or longer. AOFD-2 dives deep into atmospheric and oceanic fluid dynamics, spanning a wealth of topics while offering the reader lucid, pedagogical, and thorough presentations across a universe of knowledge. There are really three books here: one focused on geophysical fluid dynamic fundamentals; a second on atmospheric dynamics; and a third on oceanic dynamics. Each part offers new material relative to the first edition, as well as the reworking of earlier presentations to enhance pedagogy and update understanding based on recent research. . . . the reader is privileged to receive a unified presentation from a master scientific writer whose pedagogy is unmatched in the discipline. This book is a truly grand achievement. It will be well used by fluid dynamicists, oceanographers, atmospheric scientists, applied mathematicians, and physicists for decades to come. Each sentence, paragraph, section, chapter, and figure, are thoughtful and erudite, providing the reader with insights and rigor needed to truly capture the physical and mathematical essence of each topic."

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"Vallis speaks my language. He successfully weaves together fundamental theory, physical intuition, and observed phenomena to tell the story of geophysical fluid behavior at local and global scales. This multi-pronged approach makes this an ideal text for both beginners and experts alike - there is something for everyone. This is why it is the book I use for my class, the book I recommend to incoming graduate students (no matter their background) and the book I go to first when I need clarity on GFD topics. The first edition of this book has been my go-to text since it was first published . . . With the new edition, we now get an even more comprehensive view of how the fundamental processes that dictate the evolution of our atmosphere and oceans drive the complex phenomena we observe."

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Trevor J. McDougall, *University of New South Wales*

"Researchers looking for an informative and coherent treatment of the dynamics of the atmosphere and ocean, starting at a fundamental level, and proceeding to advanced topics, will find that this book is a truly superb resource. The book is particularly notable for its even-handed treatment of the ocean and the atmosphere and its synthetic discussion of observations, numerics and analytic methods."

William R. Young, *Scripps Institution of Oceanography*

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Geoffrey K. Vallis
University of Exeter

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University Printing House, Cambridge CB2 8BS, United Kingdom

One Liberty Plaza, 20th Floor, New York, NY 10006, USA

477 Williamstown Road, Port Melbourne, VIC 3207, Australia

4843/24, 2nd Floor, Ansari Road, Daryaganj, Delhi – 110002, India

79 Anson Road, #06–04/06, Singapore 079906

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www.cambridge.org
Information on this title: www.cambridge.org/9781107065505
DOI: 10.1017/9781107588417

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First published 2017

Printed in the United Kingdom by Clays, St Ives plc

A catalogue record for this publication is available from the British Library

ISBN 978-1-107-06550-5 Hardback

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The Falconer

Turning and turning in the widening gyre
W. B. Yeats.

Soaring and diving in the spiraling gyre
The freed falcon divines
Earth's air and water.

B. A. Wingate, 2017.

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In the main text, sections that are more advanced or that contain material that is peripheral to the main narrative are marked with a black diamond, ✦. Sections that contain material that is still not settled or that describe active areas of research are marked with a dagger, †.

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The sea obscured by vapour that … slid in one mighty mass along the sea shore … The distant country, overhung by straggling clouds that sailed over it, appeared like the darker clouds, seen at a great distance apparently motionless, while the nearer ones pass quickly over them, driven by the lower winds. 'I never saw such a union of earth, sky and sea. The clouds beneath our feet spread to the water, and the clouds of the sky almost joined them.

Dorothy Wordsworth, *Alfoxden Journal*, 1798.

Preface

THIS IS A BOOK ON THE DYNAMICS OF THE ATMOSPHERE AND OCEAN, with an emphasis on the fundamentals and on the large-scale circulation. By 'large-scale' I mean scales between that of the weather (a few hundred kilometres in the atmosphere and a few tens of kilometres in the ocean, which indeed has its own weather) and the global scale. My focus is our own planet Earth, for that is where we live, but the principles and methodology used should be appropriate for the study of the atmospheres and oceans of other planets. And even if we stay at home, I try to take the reader on a journey — from the most basic and classical material to the frontiers of knowledge.

The book is written at a level appropriate for advanced undergraduate or graduate courses; it is also meant to be a useful reference for researchers, and some aspects of the book have the nature of a research monograph. Prior knowledge of fluid dynamics is helpful but is not a requirement, for the fluid equations are introduced in the first chapter. Similarly, some knowledge of thermodynamics will ease the reader's path, but is not essential. On the other hand, the reader is assumed to have a working knowledge of vector calculus and to know what a partial differential equation is.

Atmospheric and oceanic fluid dynamics (AOFD) is both pure and applied. It is a pure because it involves some of the most fundamental and unsolved problems in fluid dynamics — problems in turbulence and wave–mean flow interaction, in chaos and predictability, and in the general circulation itself. It is applied because the climate and weather so profoundly affect the human condition, and the practice of weather forecasting is a notable example of a successful (yes it is) applied science. The field is also broad, encompassing such subjects as the general circulation, instabilities, gyres, boundary layers, waves, convection and turbulence. My goal in this book is to present a coherent selection of these topics so that the reader will gain a solid grounding in the fundamentals, motivated by and with an appreciation for the problems of the real world. The book is primary a theoretical one, but in a number of places observations are used to illustrate the dynamics and define the problems. And I have tried to lead the reader to the edge of active areas of research — for a book that limits itself to what is absolutely settled would, I think, be rather dry, a quality best reserved for martinis and humour.

This book is a major revision of one that first appeared in 2006, and about half of the material here is new or rewritten. Some of the changes were motivated by the fact that half of the planet lies equatorward of 30°, that about 20% of the mass of the atmosphere lies above 10 km, and that it rains. Overall, I have tried to encompass the most important aspects of the dynamics and circulation of both atmosphere and ocean, for the similarities and differences between the two are so instructive that even if one's interest is solely in one there is much to be learned by studying the other. Where the subject matter verges on areas of research, I have focused on ideas and topics that, I think, will be of lasting value rather than of current fashion; however, this choice is undoubtedly influenced by my own interests and expertise, not to mention my prejudices.

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How to Use this Book

This book is a long one, but it might help to think of it as four short ones with a common theme. These books are on the fundamentals of geophysical fluid dynamics (GFD); waves, instabilities and turbulence; atmospheric circulation; and ocean circulation. Each short book forms a Part in this book, and each might form the basis for a course of about a term or a semester, although there is more material in each than can comfortably be covered unless the course hums along at a torrid pace. The ordering of the topics follows a logical sequence, but is not the order that the book need be read. For example, Chapter 1 covers the basic equations of motion for a fluid, including the requisite thermodynamics. The thermodynamics of seawater belongs to and is included in that chapter, but this material is quite advanced and is not needed to understand many of the later chapters, and may be skipped on a first reading. Typically, sections that are more advanced or that are peripheral to the main narrative are marked with a black diamond, ✦, and sections that contain matters that are not settled or that describe active areas of research are marked with a dagger, †, but there is some overlap and arbitrariness in the markings. In some of these areas the book may be thought of as a entrance to the original literature and it gives my own view on the subject.

Putting the various topics together in one book will, I hope, emphasize their coherence as part of a single field. Still, there are many paths through the woods and the following notes may help steer the reader, although I do not wish to be prescriptive and experienced instructors will chart their own course. In a nutshell, the material of Part I forms the basis of all that follows, and subsequent Parts may then be read in any order if the reader is willing to cross refer, often to Part II. With care, one may also construct a single course that combines aspects of Parts II, III and/or IV.

A basic GFD course

A first course in GFD might cover much of Part I and, in many cases, Rossby waves and possibly baroclinic instability from Part II. If the students have already had a course in fluid dynamics then much of Chapter 1 can be skipped. Some of the thermodynamics may in any case be omitted on a first reading, and a basic course might encompass the following:

For students with no fluid mechanics:

Equations of motion, Sections 1.1–1.6, omitting starred sections.
Compressibility, Sections 1.8 and 1.9.
Energetics, Section 1.10 (optional).

Then, for all students:

Rotational effects and Boussinesq equations, Sections 2.1–2.4.
Pressure coordinates, Section 2.6 (optional, mainly for meteorologists).
Hydrostatic and geostrophic balance, Sections 2.7 and 2.8.
Static instability, Section 2.10.

Shallow water equations and geostrophic adjustment: Sections 3.1, 3.2, 3.5, 3.7–3.9.

Vorticity, Sections 4.1–4.4 (for students with no fluids background).
Potential vorticity, some or all of Sections 4.5 and 4.6.

Geostrophic theory, Sections 5.1–5.5.
Ekman layers, Section 5.7.

Rossby waves, Sections 6.4 and 6.5, and possibly Section 16.5 on vertical propagation.
Jet formation, Section 15.1.
Baroclinic instability, Sections 9.5 and/or 9.6.

One option is to use only the Boussinesq equations (and later the shallow water equations) and the beta plane, eschewing pressure co-ordinates and sphericity, coming back to them later for atmospheric applications. This option also makes the derivation of quasi-geostrophy a little easier.

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But meteorologists may well prefer to introduce these topics from the outset, and use pressure coordinates and not the Boussinesq approximation. Many first courses will end after Rossby waves, although a rapidly-moving course for well-prepared students might cover some aspects of baroclinic instability. If waves are being covered separately, then an alternative is to cover some aspect of the general circulation of the atmosphere or ocean, for example the Stommel model of ocean gyres in Section 19.1, which is a pedagogical delight.

Subsequent reading

The other three short books, or Parts to this book, are loosely based on more advanced courses that I have taught at various times, and taken together are an attempt to pull together classical ideas and recent developments in the field into a coherent whole. Each Part is more-or-less self-contained, although some aspects of Parts III and IV build on Part II, especially the sections on Rossby waves and baroclinic instability. An understanding of the essentials of geostrophic turbulence and turbulent diffusion will also help. Provided these sections and the material in Part I is mastered the remaining Parts may be read in any order if the reader is willing to flip pages occasionally and consult other sections as needed. I won't suggest any specific syllabus to follow apart from the contents of the book itself, but let me make a few remarks.

Part II: Waves, instabilities and turbulence

Rossby waves, gravity waves and baroclinic instability could form the first half of a second course on GFD, with wave–mean-flow interaction and/or turbulence the second half. Any one of these topics could be a full course, especially if supplemented from other sources. WKB methods are very useful in a variety of practical problems and can readily be taught in an elementary way. If needs be equatorial waves (Chapter 8) could be treated separately, for it is a little mathematical, and perhaps folded into a course covering equatorial circulation more generally. One might also construct a course based around geostrophic turbulence and the mid-latitude circulation of the atmosphere and/or ocean.

Part III: Atmospheric circulation

My focus here is on the large scale, and on the fundamental aspects of the general circulation — how wide is the Hadley Cell? why do the surface winds blow eastward in mid-latitudes? why is there a tropopause and why is it 10 km high and not 5 or 50 km? and so forth. A basic knowledge of GFD, such as is in the sample course above, is a pre-requisite; other sections of Part II may be needed but can be consulted as necessary. Chapters 14 and 15 form a pair and should be read consecutively, and Chapter 16 is in some ways a continuation of Chapter 5 on waves. There are numerous ways to navigate through these chapters — one might omit the stratosphere chapter, or it could form the basis for an entire course if other literature were added. The same applies to Chapter 18 on the tropics, and here especially the material cuts close to the bone of what is known and other researchers might write a very different chapter.

Part IV: Oceanic circulation

Many of the above comments on atmospheric circulation apply here, as my goal is to describe and explain the fundamental dynamical processes determining the large-scale structure and circulation of the ocean — why go gyres have western boundary currents? what determines the structure of the thermocline? why does El Niño occur? As for Part III, a basic knowledge of GFD, such as is in the sample course above, is a pre-requisite. Chapter 19 on wind-driven gyres is the foundation of much that follows, and Chapter 20 flows naturally from it. The chapters on the meridional overturning circulation and El Niño delve into active areas of research, and again others might have written differently. Both here and in my discussion of atmospheric circulation I take the position of a *practical theoretician;* such a person seeks theories or explanations of phenomena, but they should be relevant to the world about us.

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Miscellany and Acknowledgements

The book was written in LaTeX using Minion fonts for text and Minion Math from Typoma for equations. I fear that the references at the end disproportionately represent articles written in English and by British and North American authors, for that is the community I have mainly socialized with. The references are simply those with which I happen to most familiar — but this is a lazy choice and I apologize for it. Student exercises, various codes, and all of the figures, may be downloaded from the CUP website or my own website, which is best obtained using a search engine.

Many, many, colleagues and students have helped in the writing of this book, both by offering constructive suggestions and by gently and not so gently pointing out errors and misconceptions. Many thanks to all of you! I acknowledge your input in the endnotes after each chapter, although I am afraid that I have omitted many of you. If you have additional input or find mistakes, please email me. I would also like to thank three of my predecessor authors. The pioneering books by Joe Pedlosky and Adrian Gill paved the way and if those books had not existed I would have had neither the knowledge nor the courage to write mine. And the book by Rick Salmon taught me that careful arguments can be couched in plain language, and that an easy-to-read style can at the same time be clear and precise. I have tried to follow that example.

As with the first edition, this book ultimately owes its existence to my own hubris and selfishness: hubris to think that others might wish to read what I have written, and selfishness because the enjoyable task of writing such a book masquerades as work.

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NOTATION

Variables are normally set in italics, constants (e.g, π, i) in roman (i.e., upright), differential operators in roman, vectors in bold, and tensors in bold sans serif. Thus, vector variables are in bold italics, vector constants (e.g., unit vectors) in bold upright, and tensor variables are in bold slanting sans serif. A subscript denotes a derivative only if the subscript is a coordinate, such as x, y, z or t, or when so denoted in the text. A subscript 0 generally denotes a reference value (e.g., ρ_0). The components of a vector are denoted by superscripts. If a fraction contains only two terms in the denominator then brackets are not always used; thus $1/2\pi = 1/(2\pi) \neq \pi/2$.

The lists below contain only the more important variables or instances of ambiguous notation, in quasi-alphabetical order, first of Roman characters and then of mainly Greek characters and operators. Distinct meanings are separated with a semi-colon.

Variable	Description
a	Radius of Earth.
b	Buoyancy, $-g\delta\rho/\rho_0$ or $g\delta\theta/\tilde{\theta}$.
B	Planck function, often σT^4.
$\boldsymbol{c}_g$	Group velocity, (c_g^x, c_g^y, c_g^z).
c_p	Phase speed; heat capacity at constant pressure.
c_v	Heat capacity at constant volume.
c_s	Sound speed.
f, f_0	Coriolis parameter, and its reference value.
$\boldsymbol{g}$, g	Vector acceleration due to gravity, magnitude of $\boldsymbol{g}$.
g	Gibbs function.
$\mathbf{i}, \mathbf{j}, \mathbf{k}$	Unit vectors in (x, y, z) directions.
i; i	An integer index; square root of minus one.
I	Internal energy.
$\boldsymbol{k}$	Wave vector, with components (k, l, m) or (k^x, k^y, k^z).
k_d	Wave number corresponding to deformation radius.
L_d	Deformation radius.
L, H	Horizontal length scale, vertical (height) scale.
m	Angular momentum about the Earth's axis of rotation.
N	Buoyancy, or Brunt–Väisälä, frequency.
p, p_R	Pressure, and a reference value of pressure.
Pr	Prandtl ratio, f_0/N.
q	Quasi-geostrophic potential vorticity; water vapour specific humidity.
Q	Potential vorticity (in particular Ertel PV).
$\dot{Q}$	Rate of heating.
Ra	Rayleigh number.
Re ; Re	Real part of expression; Reynolds number, UL/ν.
Ro	Rossby number, U/fL.
S	Salinity; source term on right-hand side of an evolution equation.
$S_o, \mathbf{S}_o$	Solenoidal term, solenoidal vector.
T	Temperature; scaling value for time.
t	Time.
$\boldsymbol{u}$	Two-dimensional (horizontal) velocity, (u, v).
$\boldsymbol{v}$	Three-dimensional velocity, (u, v, w).
w	Vertical velocity; water vapour mixing ratio.
x, y, z	Cartesian coordinates, usually in zonal, meridional and vertical directions.
Z	Log-pressure, $-H \log p/p_R$; scaling for z.

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Variable	Description
$\mathcal{A}$	Wave activity.
α	Inverse density, or specific volume; aspect ratio.
β; β^*	Rate of change of f with latitude, $\partial f/\partial y$; $\beta^* = \beta - u_{yy}$
$\beta_T, \beta_S, \beta_p$	Coefficient of expansion with respect to temperature, salinity and pressure, respectively.
ϵ	Generic small parameter (epsilon).
ε	Cascade or dissipation rate of energy (varepsilon).
η	Specific entropy; perturbation height; enstrophy cascade or dissipation rate.
$\boldsymbol{\mathcal{F}}$	Eliassen Palm flux, $(\mathcal{F}^y, \mathcal{F}^z)$.
γ	The ratio c_p/c_v; Vorticity gradient, e.g., $\beta - u_{yy}$.
Γ	Lapse rate (sometimes subscripted, e.g., Γ_z, but here this does not denote a differential).
κ	Diffusivity; the ratio R/c_p.
$\mathcal{K}$	Kolmogorov or Kolmogorov-like constant.
Λ	Shear, e.g., $\partial U/\partial z$.
μ	Viscosity; chemical potential.
ν	Kinematic viscosity, μ/ρ.
υ	Meridional component of velocity.
$\mathcal{P}$	Pseudomomentum.
ϕ	Pressure divided by density, p/ρ.
φ	Passive tracer.
Φ	Geopotential, usually gz; scaling value of ϕ.
Π	Exner function, $\Pi = c_p T/\theta = c_p(p/p_R)^{R/c_p}$; an enthalpy-like quantity.
$\boldsymbol{\omega}$	Vorticity.
$\Omega, \boldsymbol{\Omega}$	Rotation rate of Earth and associated vector.
ψ	Streamfunction.
ρ	Density.
ρ_θ	Potential density.
σ	Layer thickness, $\partial z/\partial\theta$; Prandtl number ν/κ; measure of density, $\rho - 1000$.
$\boldsymbol{\tau}$	Stress vector, often wind stress.
$\widetilde{\boldsymbol{\tau}}$	Kinematic stress, $\boldsymbol{\tau}/\rho$.
τ	Zonal component or magnitude of wind stress; eddy turnover time; optical depth.
θ; Θ	Potential temperature; generic thermodynamic variable, often conservative temperature.
ϑ, λ	Latitude, longitude.
ζ	Vertical component of vorticity.
$\left(\dfrac{\partial a}{\partial b}\right)_c$	Derivative of a with respect to b at constant c.
$\left.\dfrac{\partial a}{\partial b}\right\|_c$	Derivative of a with respect to b evaluated at $b = c$.
∇_a	Gradient operator at constant value of coordinate a. Thus, $\nabla_z = \mathbf{i}\partial_x + \mathbf{j}\partial_y$.
$\nabla_a\cdot$	Divergence operator at constant value of coordinate a. Thus, $\nabla_z\cdot = (\mathbf{i}\partial_x + \mathbf{j}\partial_y)\cdot$.
$\nabla^\perp$	Perpendicular gradient, $\nabla^\perp\phi \equiv \mathbf{k}\times\nabla\phi$.
curl_z	Vertical component of $\nabla\times$ operator, $\mathrm{curl}_z\boldsymbol{A} = \mathbf{k}\cdot\nabla\times\boldsymbol{A} = \partial_x A^y - \partial_y A^x$.
$\dfrac{\mathrm{D}}{\mathrm{D}t}$	Material derivative (generic).
$\dfrac{\mathrm{D}_g}{\mathrm{D}t}$	Material derivative using geostrophic velocity, for example $\partial/\partial t + \boldsymbol{u}_g\cdot\nabla$.

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